Defense Technical Information Center Compilation Part Notice

ADP010928

TITLE: Revealing the Multifractal Nature of Failure Sequence

DISTRIBUTION: Approved for public release, distribution unlimited

This paper is part of the following report:

TITLE: Paradigms of Complexity. Fractals and Structures in the Sciences

To order the complete compilation report, use: ADA392358

The component part is provided here to allow users access to individually authored sections of proceedings, annals, symposia, ect. However, the component should be considered within the context of the overall compilation report and not as a stand-alone technical report.

The following component part numbers comprise the compilation report:
ADP010895 thru ADP010929

REVEALING THE MULTIFRACTAL NATURE OF FAILURE SEQUENCE

R. I. ZAINETDINOV

Moscow State University of Railway Communication, 15 Obraztsov Street, Moscow, 101475, Russia
E-mail: zri@hotmail.com

The purpose of the paper is to introduce a new application of the multifractals in the reliability engineering, risk analysis, historical chronology and others fields where we deal with series of events of various natures. The spectrum of such series is quite broad. In particular, we do not know whether some temporal pattern is hidden in an apparently disordered set of events. The multifractal theory is a good basis for revealing such an order and describing the event sequence in time. It can provide a deeper understanding the nature of the event flow.

A mathematical construction that represents an event sequence as a set of the random points on the time scale is referred to as a stochastic point process. It can either be modeled as a list of impulses located at times where events occur or as a count process. Let S be a sample of events of limited size N_0 during the specified period of time $[0, \tau_{max}]$. The process time history for the sample S is represented by a sequence of idealized impulses of vanishing width, located at specified moments of the event time τ_i, i = 1, 2,..., N_0. Further, let us rescale the time τ_i of every i-th member of sample S on the maximum value τ_{max} $t_i = \tau_i/\tau_{max}$, so we can consider the event distribution on the unit interval of time T = [0, 1]. In order to characterize this distribution we divide the unit interval into temporal subintervals of duration $\Delta t = 2^{-n}$. So $N = 2^n$ subintervals are needed to cover interval T, where n is the number of generation in the binary subdivision of the temporal interval T. The distribution of the sample population over the temporal interval is specified by the numbers, N_j, of members of the sample S in the j-th subinterval. We use the fraction of the total population $\mu_j = N_j/N_0$ as a probabilistic measure for the content in subinterval Δt_j. The set of such measures presents a complete description of the event's distribution on the unit temporal interval T at stated resolution Δt [3].

Now let us consider a case that satisfies the Bernoulli trial conditions. In our interpretation an event of interest is the failure occurred on the first half of temporal interval with probability *p*. The series of Bernoulli trials with parameter *p* is a sequence of independent trials in which there are only two outcomes, and probability *p* remains the same for all generations of the binary subdivision process of interval T. Three cases associated with typical form of cumulative distribution function are considered: process with early events (when $p>0.5$), Poisson process ($p=0.5$), and process with late events ($p<0.5$). In the case of Bernoulli trials the measure μ is recursively generating by a multiplicative binomial process. The binomial cascade provides an example of a probability, which has a rich asymptotic structure and is, in modern terms, multifractal [3]. Connection between parameter *p* of the Bernoulli trial and multifractal spectrum is considered.

For the approbation of the technique, a computer simulation study has been done. As a first step, we have carried out a multiscale analysis of data generated by Bernoulli trials. The binomial multiplicative process produces shorter and shorter temporal subintervals Δt that contain less and less fractions of the total measure and generates a multifractal probabilistic measure, supported by Cantor set on the unit temporal interval.

In order to verify the fractality of data sets obtained from numerical simulations, tests and inspections of real devices a wavelet analysis was carried out. The analysis shows that maxima lines of the continuous wavelet transform (CWT) coefficients are converging towards the singularities of the measure, and they reproduce its hierarchical structure [1]. The successive forkings occurred at different scales reveal the multifractal nature of measure. Failure occurrence is probabilistic process, which results in the formation of self-similar, or rather self-affine temporal clusters. Overwhelming evidence from computer simulation of different measures indicates that these patterns are self-affine fractals, meaning that their complication is the same at different scales of observation.

The wavelet analysis was carried out by using the WaveLab package [2].[a] The multifractal structures proposed in the sequence of events are real-time structures, in contrast to fractal attractors, which reside in phase space. Thus the wavelet transform can be applied directly to a series of statistical data on reliability obtained from experiments and inspections of technical state under real service conditions [4]. The graph of the CWT coefficients of the failure time history shows that the successive forkings produce a multifractal temporal structure. Increasing the magnification of the wavelet transform microscope reveals progressively the successive generations of branching. The symmetry of the graph is broken by non-uniformity of probabilistic measure. Let $N(a)$ be the number of maxima lines in the CWT skeleton at the scale a. In the limit, as the scale a tends to 0, the ratio $\ln(N(a)/\ln(a)$ is associated to the exponent α [1]. The concentration of data points around the straight line observed in the plot of $\ln(N(a))$ versus $\ln(a)$ can be regarded as a quantitative indication of the self-similarity of the event sequence in real data sets. The ratio between the time scales of successive generation can take different values, which is another indication that a multifractal description is appropriate. Wavelet analysis of empirical data on reliability provides probabilistic evidence for the existence of a multiplicative process hidden in the temporal ordering of the failure sequence.

The plots of empirical cumulative distribution function of the lifetime data have an evident feature, namely, they are constant almost everywhere except in those points where failures occur. In the limit, the empirical distribution function resembles in a sense a devil staircase prefractal. In the proposed multifractal approach the emphasis switches to letting the statistical data "speak for themselves", rather than approximating the lifetime distribution by one of the parametric models. In practice, we are often interested in prediction of the mean time of a failure-free operation as well as the others reliability indexes. Deeper insight into failures, their prediction and prevention is to be gained by using the multifractal approach in the reliability engineering.

References

1. Arneodo A., Wavelet analysis of fractals: from the mathematical concepts to experimental reality. In *Wavelets. Theory and Applications*, ed. by G. Erlebacher, Y. Hussaini, and L. Jameson (Oxford University Press, New York, 1996) pp. 349-502.
2. Buckheit J., and Donoho D., WaveLab and Reproducible Research. In *Wavelets and Statistics*, ed. by A. Antoniadis and G. Oppenheim (Springer, New York, 1995) pp. 55-81.
3. Feder J., *Fractals* (Plenum Press, New York, 1989).
4. Zainetdinov R. I., Wavelet Analysis of Statistical Data on Reliability for Exploring the Multifractal Properties of Failure-Cascading Process. In *Wavelets and Multiscale Methods* (INRIA, Paris, Tangier, 1998).

[a] The author would like to thank the contributors of the WaveLab package.